Interferences

Cooperative effects: superradiance…

Strong links between cooperativity and dipole – dipole interactions

What is the cooperative behavior?

- It concerns the collective behavior of an assembly of *N* atoms (molecules or any emitters) versus the electromagnetic field
- The assumption is that the *N* emitters are indiscernible versus the e.m. field
- It has been introduced by R. H. Dicke in 1954 with the cooperative spontaneous emission or superradiance by an assembly of *N* atoms
- The concept of cooperative behavior can be extended to the absorption of the e.m. by *N* atoms (or emitters)

Cooperative spontaneous emission

Superradiance and subradiiance

Dicke superradiance or cooperative spontaneous emission

- Dicke considers *N* excited atoms contained in a volume, the dimensions of which are small compared to the wavelength of emitted light
- The atoms are therefore indiscernible versus the e.m. field
- If we assume that each atom is described by a two-level system, with an electric dipole equivalent to a spin ½
- The *N* atoms can be described by the collective kinetic momenta, sum of the *N* spin ½
- The superradiance corresponds to the spontaneous decay of a giant dipole operator $j = N/2$ (emission of a delayed pulse with a maximum intensity scaling as N^2 and a width as $1/N$)

N two-level (1,2) atoms (*i*)

$$H = H_R + \sum_{i=1}^{N}\left(H_{Ai} + H_{RAi}\right)$$

$$H_R = \sum_{kl} hcka_{kl}^{+}a_{kl} \;\; ; \;\; H_{Ai} = \hbar\omega_0 \left|2,i\right\rangle\left\langle 2,i\right| \;\; ; \;\; H_{RAi} = -\vec{E}.\vec{\mu}_i$$

$$\vec{E} = i\sum_{kl}\sqrt{\frac{hck}{2\varepsilon_0 L^3}}\, a_{kl}\vec{e}_l \exp\left(i\vec{k}.\vec{x}_i\right) + h.c.$$

$$\vec{\mu}_i = \vec{\mu}_i^{+} + \vec{\mu}_i^{-} \;\; ; \;\; \vec{\mu}_i^{+} = \left\langle 2\left|\vec{\mu}\right|1\right\rangle \left|2,i\right\rangle\left\langle 1,i\right| = \left\langle 2\left|\vec{\mu}\right|1\right\rangle r_i^{+} \;\; ; \;\; \vec{\mu}_i^{-} = \left(\vec{\mu}_i^{+}\right)^{+}$$

Master equation in Born-Markov approximation (interaction representation)

$$\frac{d\sigma}{dt} = -\frac{\Gamma}{2}\sum_{i=1}^{N}\left(\left[r_i^{+}r_i^{-},\sigma(t)\right]_{+} - 2r_i^{-}\sigma(t)r_i^{+}\right) - \frac{1}{i\hbar}\sum_{j\neq i}\left[\vec{E}_j^{+}.\vec{\mu}_i^{-}\sigma(t) - \sigma(t)\left(\vec{E}_j^{+}\right)^{+}.\vec{\mu}_i^{-}\right]$$

$\vec{E}_j^{+}$ electric dipole field creared by j in i

$$\vec{E}_j^{+} = \frac{1}{4\pi\varepsilon_0}\left\{k_0^2\left(\frac{\vec{x}_{ij}}{x_{ij}}\times\vec{\mu}_i^{+}\right)\times\frac{\vec{x}_{ij}}{x_{ij}}\frac{e^{ik_0x_{ij}}}{x_{ij}} + \left[3\frac{\vec{x}_{ij}}{x_{ij}}\left(\frac{\vec{x}_{ij}}{x_{ij}}.\vec{\mu}_i^{+}\right) - \vec{\mu}_i^{+}\right]\left(\frac{1}{x_{ij}^3} - \frac{ik_0}{x_{ij}^2}\right)e^{ik_0x_{ij}}\right\}$$

Small volume: $K_0 x_{ij} < 1$

Master equation $$\frac{d\sigma}{dt} = -\frac{\Gamma}{2}\sum_{i=1}^{N}\left(\left[r_i^+ r_i^-, \sigma(t)\right]_+ - 2r_i^- \sigma(t) r_i^+\right)$$

$$-\frac{\Gamma}{2}\sum_{j\neq i}\left(\left[r_i^+ r_j^-, \sigma(t)\right]_+ - 2r_i^- \sigma(t) r_j^+\right) + \frac{1}{i\hbar}\sum_{j\neq i}\left[H_{dd}^{(i,j)}, \sigma(t)\right]$$

We have the terms of cooperative spontaneous emission (superradiance) and those cooresponding to dipole-dipole interaction ($H_{dd}^{(i,j)}$)

$$H_{dd}^{(i,j)} = \frac{\mu_i \mu_j}{4\pi\varepsilon_0}\left\{\left(1 - 3\cos^2\theta_{ij}\right)\frac{1}{x_{ij}^3} - \frac{\left(1+\cos^2\theta_{ij}\right)}{2}\frac{k_0^2}{x_{ij}}\right\}$$

We have the classical term of dipole-dipole interaction

+ another term a priori much smaller when $k_0 x_{ij} << 1$

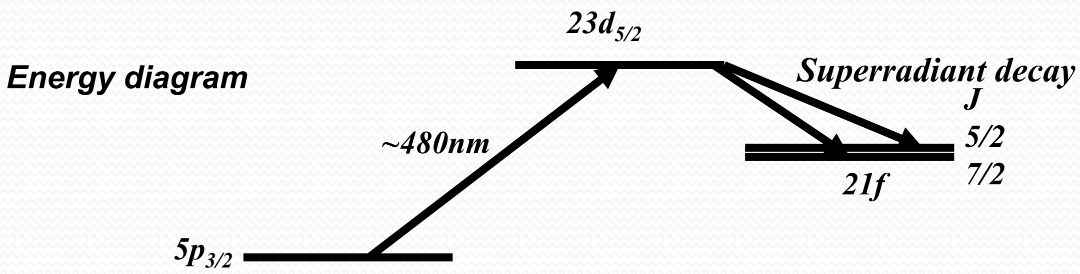
$\omega_0 : 1/(2n^3)\ a.u.\ ;\ k_0^{-1} = \mathrm{D} = \lambda / 2\pi = 115\mu m\ (n=20),\ 390\ (n=30),\ 930\ (n=40),$ $1.8mm\ (n=50),\ 3\ (n=60),\ 7.5\ (n=80),\ 14.5\ (n=100)$

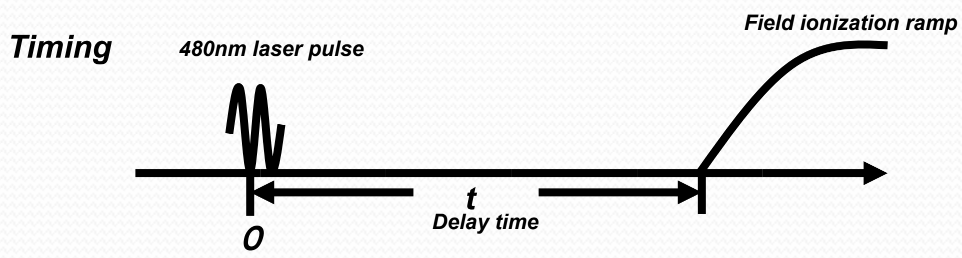
Superradiance

- Because of the dipole-dipole interactions terms, we have no cooperative emission in a small volume at least in disordered medium (the dipole emittors are no longer locked in phase)
- Dicke has also introduced the superradiance in a large pencil-shape volume where the spontaneous emission occur in one mode and where the effects of the propagation have to be taken into account

Superradiance in cold Rydberg gases (Virginia unpublished experiment: Rb)

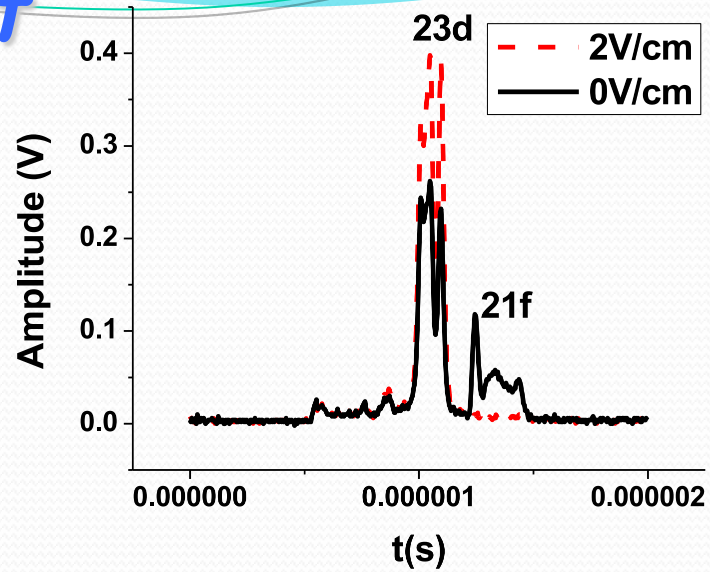
- Superradiance: cooperative emission by a large number of excited atoms. It is coherent radiation emiited with a delay varying as N^{-1} and with an intensity varying as N^2 dependence.

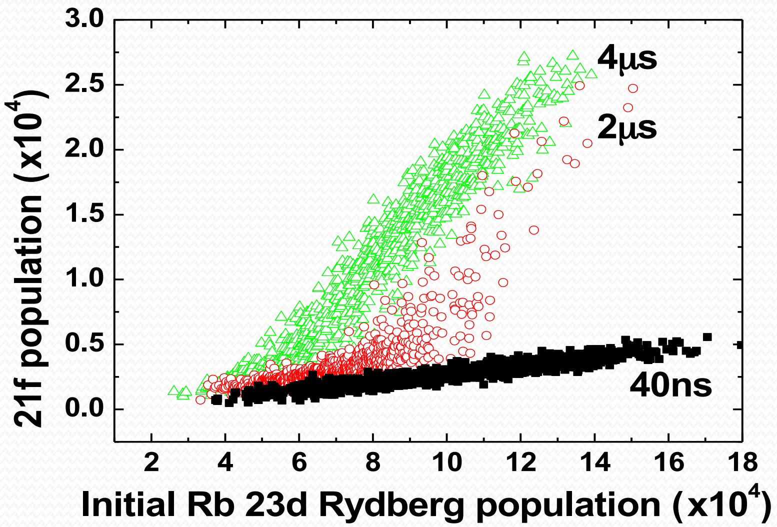


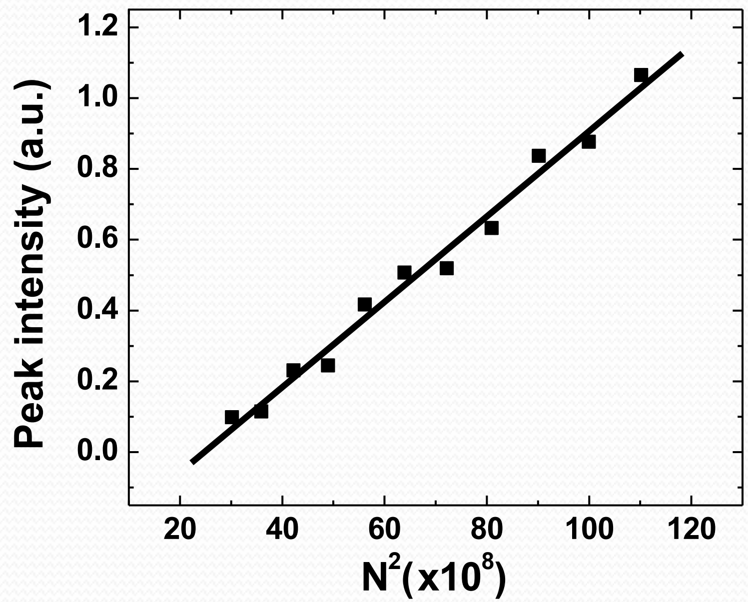


Observation of an efficient transfert after 400 ns

- Superradiance is sensitive to electric field
- Threshold behavoir
- Intensity as N^2
- We are not in the Dicke conditions
- The propagations effects have been considered







Superradiance

- Superradiance as predicted by Dicke (volume << $(\lambda/2\pi)^3$) does not occur
- The dipole-dipole interactions make that the dipole emitters are not locked in phase
- Superradiance occurs in a relative large volume compared to the emitted wavelength. The propagation effects play a role (similar to the pencil shaped experiments)
- What about an ordered medium or an ensemble of entangled atoms?
- Superradiance is still a challenge!

Dipole – dipole interactions Dipole blockade of the Rydberg excitation

Introduced by Jaksch et al. PRL, 84, 4232 (2000)for the realization of scalable quantum gates

Correlated ensembles

Dipole blockade of the Rydberg excitation induced by a static electric field

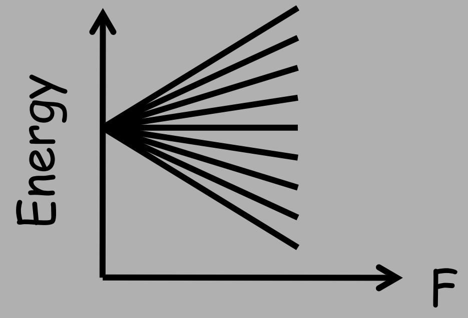
Vogt *et al.*, PRL <u>99</u>, 073002 (2007)

Dipole induced by electric field

- Stark-Rydberg manifold states have permanant dipole (High *l*)

$$\mu = -\frac{3}{2}n\left(n_1 - n_2\right)\ ,\ \mu_{\max}\ :\ \pm\frac{3}{2}n^2$$

$$n = n_1 - n_2 + |m| + 1$$

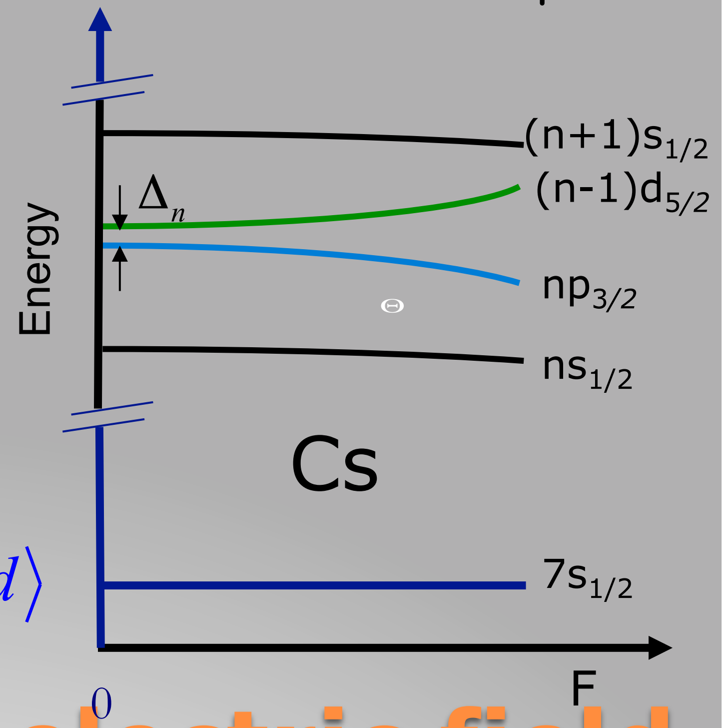


- *p* and *d* strongly Stark-coupled

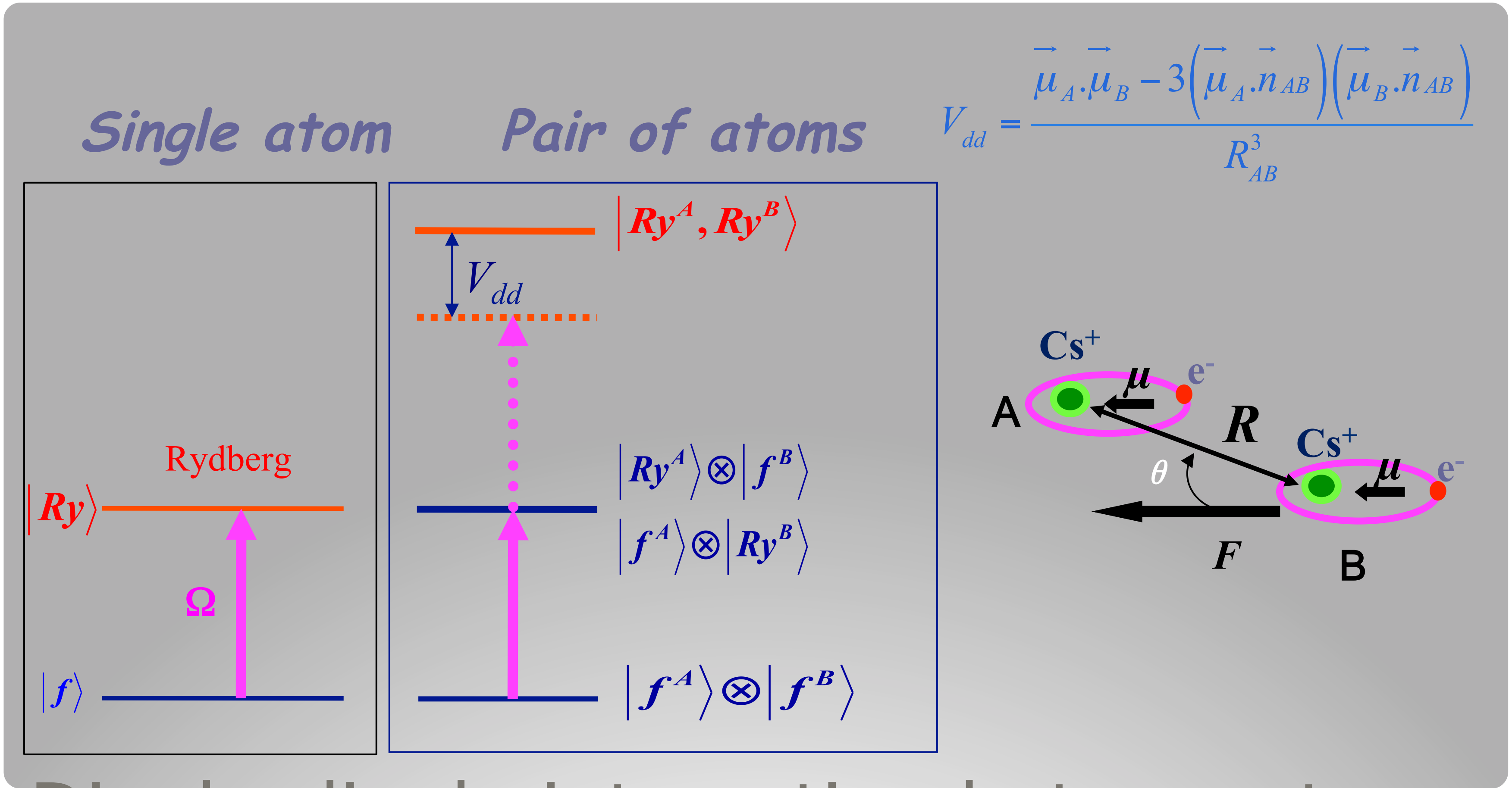
$$\mu \approx \mu_{dp}\sin\theta\quad ,\quad \mu_{dp} \sim n^2 (a.u.)$$

$$\tan\left(\theta\right) = \frac{|W_n|}{\Delta_n / 2}\quad ,\quad W_n = -\mu_{dp}F_z$$

$$|np, F\rangle \approx \cos\left(\frac{\theta}{2}\right)|np\rangle - \sin\left(\frac{\theta}{2}\right)|(n-1)d\rangle$$

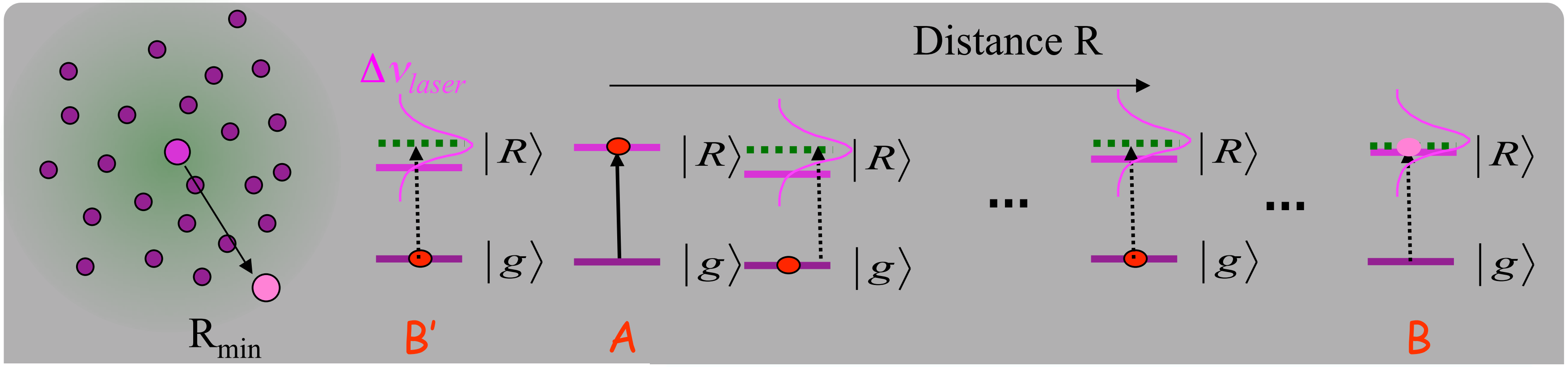


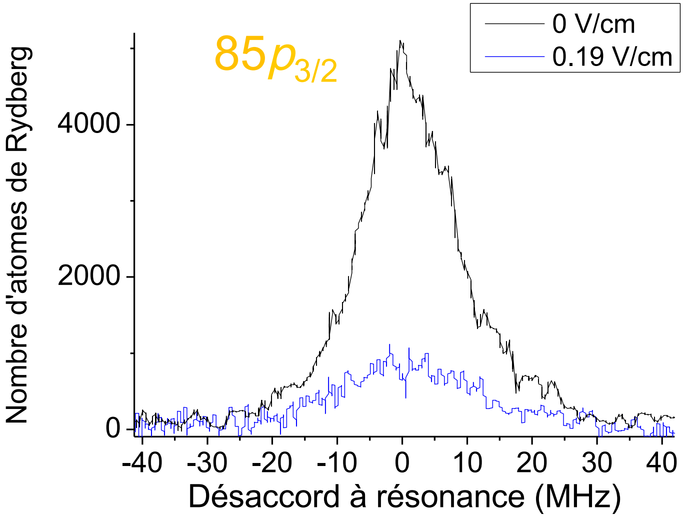
n = 30 μ_{max} ~ 3500 Debye ; n = 100 μ_{max} ~ 38000 Debye

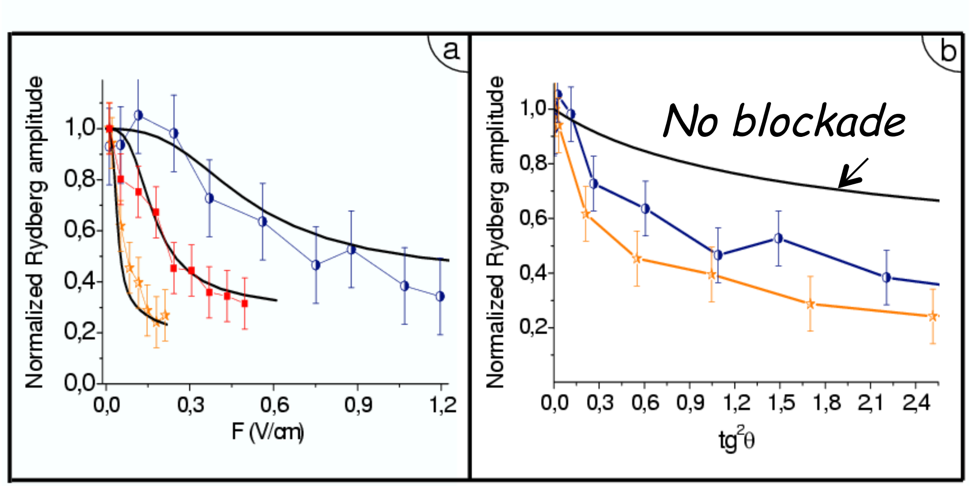


Dipole-dipole interaction between two atoms

Dipole blockade of the excitation / Conditionnal excitation







60, 70 and 85 $p_{3/2}$

Dipole blockade in an atomic sample

Spectraly broadband excitation: band of levels
Limitation of the high-resolution excitation
Role of the nearest neighbor

$$\text{Recall} : \tan(\theta) = \frac{|W_n|}{\Delta_n / 2}$$

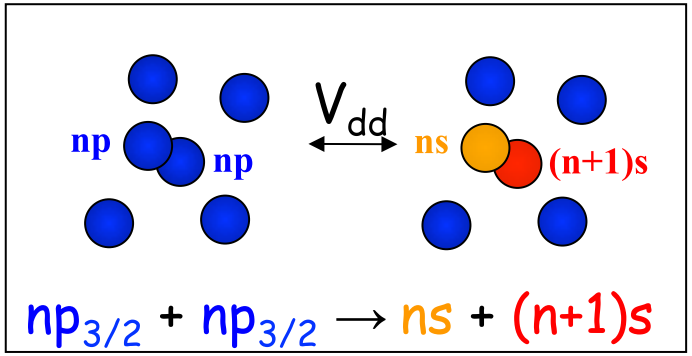
Dipole blockade of the Rydberg excitation at Förster resonance

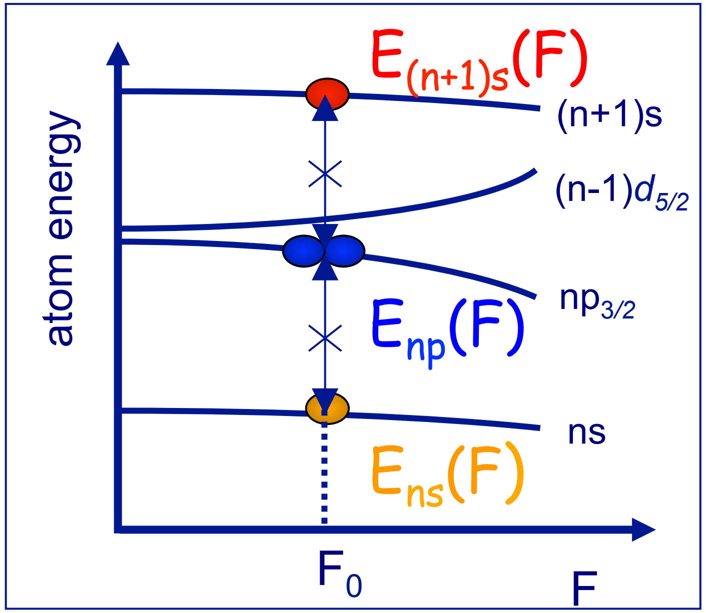
Vogt *et al.*, PRL 97, 083003 (2006)

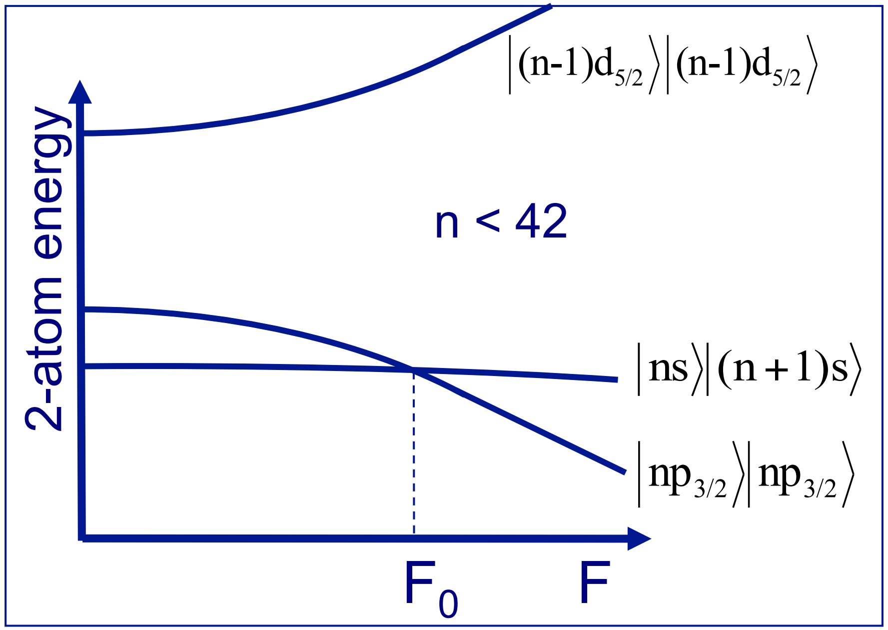
Föster resonances – dipole dipole interaction

Resonance at F_0

Cs: $2\,E_{np}(F_0) = E_{ns}(F_0) + E_{(n+1)s}(F_0)$







FRET (Förster resonance energy transfer)

Föster resonances – dipole dipole interaction

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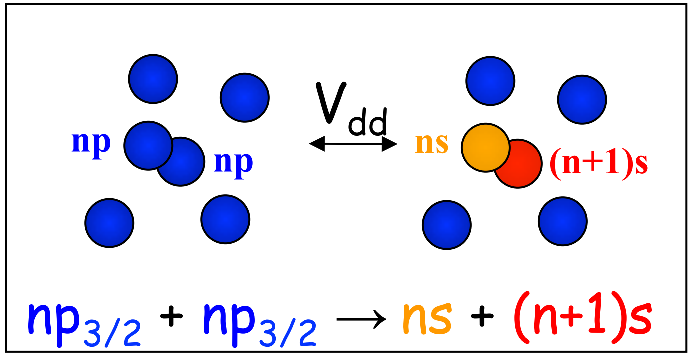
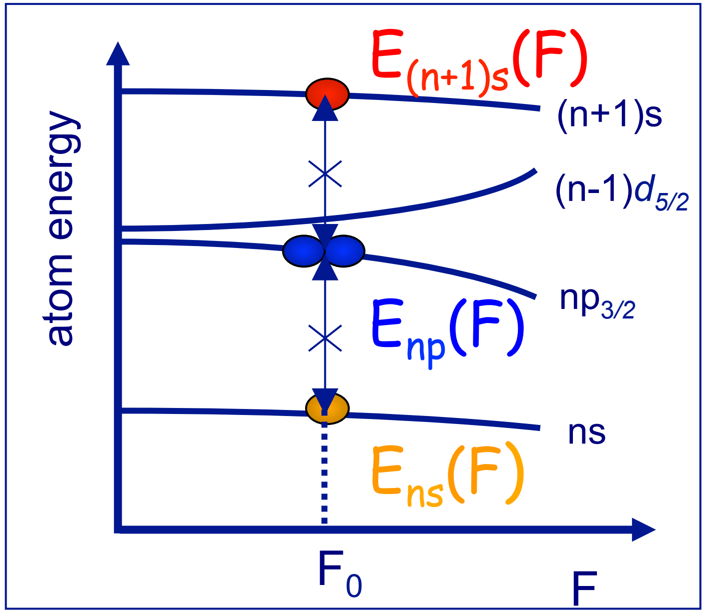
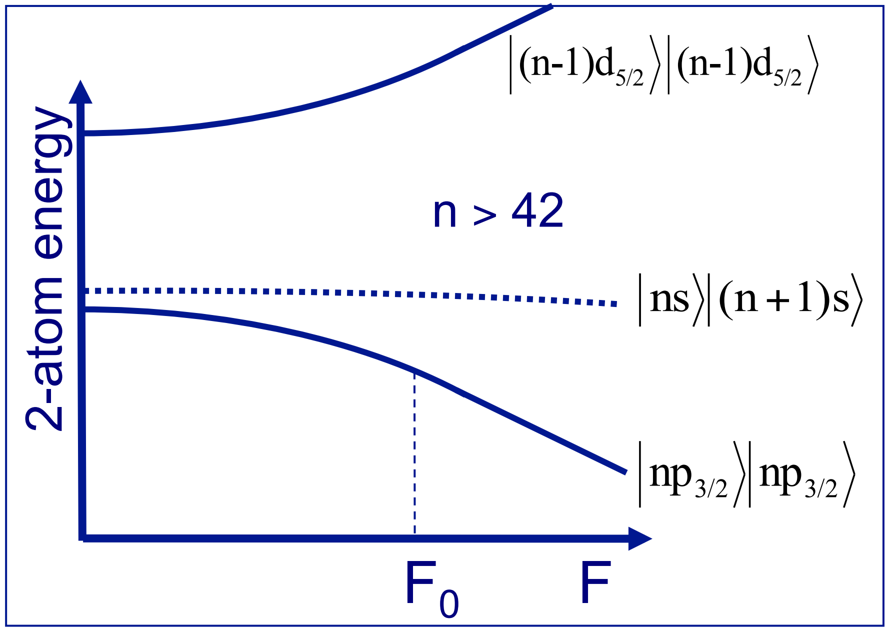
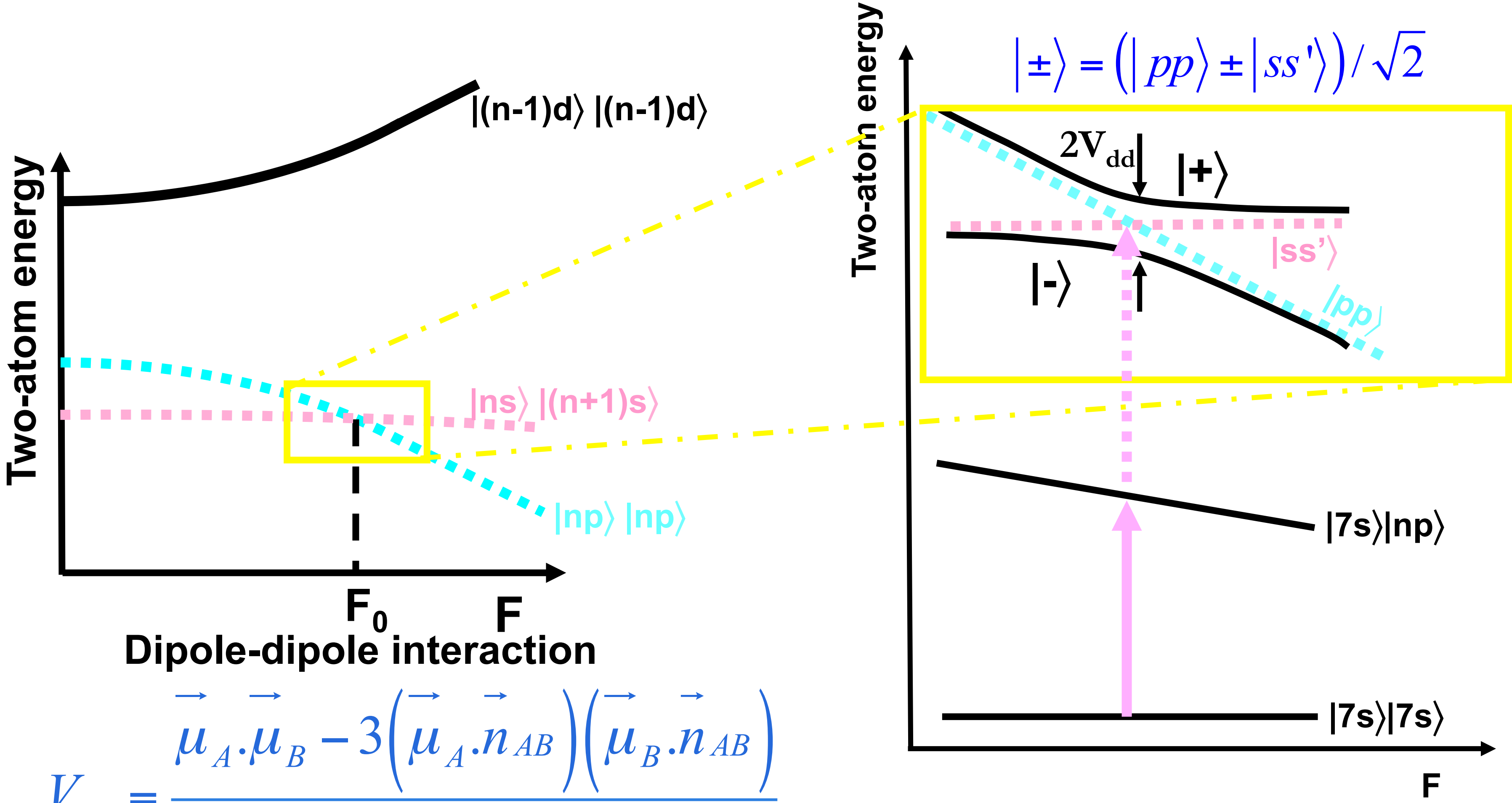






FRET (Förster resonance energy transfer)

Excitation of a pair of atoms at a Förster resonance



$$V_{dd} = \frac{\vec{\mu}_A.\vec{\mu}_B - 3\left(\vec{\mu}_A.\vec{n}_{AB}\right)\left(\vec{\mu}_B.\vec{n}_{AB}\right)}{R_{AB}^3}$$

Dipole blockade of the high-resolution Rydberg excitation ($36p_{3/2}$)

Vogt et al. PRL **97** 083003 (2006)